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Measuring Mathematics Engagement Anxiety

a MARS-style instrument for an active and interactive classroom

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September 2018

Abstract We present a validation study of fifteen new math anxiety scale items designed to augment the widely-used Revised Mathematics Anxiety Scale (RMARS). While the RMARS and other standard instruments measure students' anxiety in response to computation, test situations, and math course activities such as buying a textbook or watching a lecture, the new items address students' anxiety in response to doing mathematics in an active and interactive classroom. With a survey sample of 132 future teachers enrolled at colleges and universities, we use exploratory and confirmatory factor analyses to groups the fifteen new items into three new dimensions of math anxiety: Problem-Solving Anxiety, Explanation Anxiety, and Explanation with Internal Doubt Anxiety. Further, Cronbach's alpha for the overall scale, as well as for each dimension individually, are all between 0.9 and .95, indicating internal consistency.

1 Introduction

Researchers have long been concerned with measuring math anxiety and its effects on math performance through emotional, physiological, or behavioral mechanisms (e.g., Novak & Tassell, 2017; Ashcraft & Moore, 2009; Ashcraft & Krause, 2007; Ashcraft, 2002; Ashcraft & Kirk, 2001; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998; Hembree, 1990). As mathematicians and college math instructors, our interest in math anxiety centers less on physiological and behavioral manifestations and more on the socio-cultural practice of doing mathematics in our classrooms. For example, *how does math anxiety affect students' ability to participate in our classrooms? And what can we do about it? How might students' math anxiety inform our assessment decisions? Which practices of doing mathematics induce anxiety in our students?* For

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students who are training to be teachers, *how does anxiety around math practices in our classrooms relate to the math practices they choose to implement (or not) in their own future classrooms?* To facilitate the study of these kinds of questions, our overall goal in this paper is to put forth additional, validated items for measuring math anxiety that further illuminate the multidimensionality of the construct and allow for more nuanced studies of its effects, particularly in a student-centered mathematics classroom. The items we developed are applicable to a broad range of undergraduate mathematics settings, and especially to pre-service teachers.

In 1972, the psychologists Richardson & Suinn defined math anxiety as

> a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations. (p. 551)

This definition was given alongside a 98-item inventory to measure the construct: the Mathematics Anxiety Rating Scale, or MARS. Currently, the most commonly used instruments for measuring math anxiety are revisions of this 1972 scale (Alexander & Martray, 1989; Ferguson, 1986; Plake & Parker, 1982; Resnick, Viehe, & Segal, 1982). These all consist of Likert-scale questions that ask respondents to rate their level of anxiety in various situations on a scale from 1 ("not at all") to 5 ("very much"). Using various versions of the MARS instrument, multiple studies have found the math anxiety construct to be multidimensional (see Table 1). While it is well-established what aspects of math anxiety the MARS-based inventories measure, an examination of the items shows that they fail to capture the full spectrum either of what it means to do "solve mathematical problems" or of mathematical "academic situations" students are likely to encounter today. As trained mathematicians and involved in post-secondary mathematics pedagogy, the authors feel particularly qualified to address this issue[1].

To demonstrate these disciplinary and pedagogical differences, Figure 1 presents some example items from an existing instrument—the Revised Mathematics Anxiety Rating Scale, or RMARS (Alexander & Martray, 1989)—in comparison to some of our newly proposed items. The RMARS items rely on assumptions that

MARS/RMARS Items

- Watching a teacher work an algebraic equation on the blackboard.
- Thinking about an upcoming math test one day before.
- Listening to a lecture in math class.
- Being given a set of division problems to solve.

New (MEARS) Items

- Being asked to solve a math problem when you are not sure which formulas to use.
- Being asked to further justify why your mathematical solution is correct to a classmate who is not yet convinced.
- Sharing your solution with a small group of classmates when you are not sure it is correct.

Fig. 1 Selected items from the (Revised) Mathematics Anxiety Rating Scale (R/MARS) and from the new Mathematics Engagement Anxiety Rating Scale (MEARS).

[1] To our knowledge, no mathematicians have been involved in the creation of any of the other instruments designed to measure math anxiety.

(a) mathematics as a discipline consists of rote computations and procedures, and (b) that a mathematics classroom is centered on lectures and exams. Our items, in contrast, attempt to add a richer picture of mathematics that includes (a') solving novel problems, including those for which you do not know where to start and (b') more diverse and progressive classroom situations, including giving mathematical justifications and explanations to peers.

In this paper, we first examine the definition of math anxiety (see especially §2.2), focusing on contemporary interpretations of "solving mathematical problems" and "academic situations". Based on this literature review and our own experiences teaching college mathematics, we propose additional items (see §3.1) to augment the existing most-used instrument for measuring math anxiety, the aforementioned RMARS. Our new 15-item inventory—the Mathematics Engagement Anxiety Rating Scale (MEARS)—was written with two constructs and five sub-constructs in mind. The two main constructs we intended to measure are *Problem Solving Anxiety* (a feeling of anxiety in response to encountering conceptually difficult or novel math problems, or from the length of problems), and *Explanation Anxiety* (a feeling of anxiety in anticipation of or response to explaining one's mathematical ideas to others). The central sections of the paper report a factor analysis and validation of these new items (see Methods and Results in §3 and §4), based on data from 132 pre-service teachers at colleges and universities. Exploratory and confirmatory factor analysis of this data grouped the 15 MEARS items into three factors: our original two constructs of Problem Solving Anxiety and Explanation Anxiety and, separately, Explanation with Internal Doubt Anxiety—originally designed as a sub-construct of Explanation Anxiety[2]. Further exploratory factor analyses show that these factors are distinct from those in the Revised Mathematics Anxiety Rating Scale (RMARS) instrument, and that they are also distinct from general anxiety as measured by the State-Trait Anxiety Inventory (STAI).

2 Literature Review

The bulk of this Literature Review (§2.2) is dedicated to digging deeper into two key phrases in the definition of math anxiety (see page 2)—"solving of mathematical problems" and "academic situations"—and how our understanding of these terms should inform the way we measure math anxiety. Before dissecting this definition, we start with an overview of how math anxiety has traditionally been measured and what dimensions of math anxiety have repeatedly been identified (§2.1). These two pieces—the dimensionality of math anxiety and a deeper look into the subtleties of its definition—frame the current study's addition of new dimensions to our understanding of math anxiety. Finally, since we piloted our new instrument on a very specific population—future elementary teachers—we outline some of the literature related to math anxiety in this special population and why it is of particular interest (§2.3).

[2] These sub-constructs are described in Section 3.1.

2.1 A Short History of Measuring Math Anxiety

In a seminal 1972 paper, Richardson and Suinn developed a general purpose instrument for assessing math anxiety: the 98-item Mathematics Anxiety Rating Scale (MARS)[3]. In this survey, respondents rate the amount of anxiety they feel in various situations on a Likert scale from 1 ("not at all") to 5 ("very much"). These scenarios range from everyday life (e.g., "reading a cash register receipt after your purchase") to academic (e.g., "realizing you have to take a certain number of math classes to fulfill requirements") to classroom-specific (e.g., "watching a teacher work on an algebraic equation on the blackboard"). Due to its objectivity, availability, and reliability, it (and its revisions) became the gold standard for measuring math anxiety.

Though the authors originally claimed their instrument was unidimensional[4], subsequent studies of the MARS instrument showed that it has multiple dimensions. In one widely used version, a shortened 25-item instrument called the Revised Mathematics Anxiety Rating Scale (RMARS), Alexander & Martray identified three factors: *Math Test Anxiety*, pertaining to evaluative situations in mathematics; *Numerical Task Anxiety*, pertaining to basic math computation such as multiplication and addition; and *Math Course Anxiety*, pertaining to being in a mathematics course (Alexander & Martray, 1989). Similar factors were identified or confirmed in other studies, using different subsets of MARS items or different sample population (Baloglu & Zelhart, 2007; Bowd & Brady, 2002; Plake & Parker, 1982; Rounds & Hendel, 1980). Although various authors call similar factors by different names, we retain these factor labels from the RMARS because they capture most of the factor structure identified or confirmed in other studies.

While this three-factor structure (or something similar to it) has been confirmed repeatedly on the MARS and its variations, several authors have suggested that these are not the *only* three factors or subconstructs of math anxiety that are worth considering. According to Bessant (1995),

> The MARS has proven a reliable measure of some dimensions of mathematics anxiety, but it does not encompass the entire range of meanings implicit in this concept. Despite the multidimensional character of the MARS, items could be appended to the scale that tap additional components or themes. (p. 328)

Without adding any new items, Bessant (1995) conducted a factor analysis ($N = 173$) on a reduced version of the MARS (80 items) and found a six-factor structure. He labeled his factors General Evaluation Anxiety, Everyday Numerical Anxiety, Passive Observation Anxiety, Performance Anxiety, Mathematics Test Anxiety, and Problem-Solving Anxiety (with eigenvalues 28.09, 6.71, 3.09, 2.84, 1.78, and 1.45, respectively. He included three factors that were quantitatively less significant under the claim that "analysis of peripheral factors within the MARS provides insight into additional features of [the] complex construct [of math anxiety]" (p. 336). That is, he claims that math anxiety is more complex than the three factors so many other authors have identified.

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[3] For a more thorough history of this instrument, we refer the reader to Ashcraft & Moore, 2009.

[4] This was because they found a high alpha coefficient (.97). They claimed this indicated (1) high internal reliability and (2) items are heavily dominated by a single homogeneous factor. However, the latter is not a correct conclusion for a high alpha coefficient (Schmitt, 1996).

Phobos Scale Items
- Having to work a math problem that has x's and y's instead of 2's and 3's.
- Be asked to discuss the proof of a theorem about triangles.
- Listening to a friend explain something they have just learned in calculus.

Fig. 2 Selected original items from the Phobos scale.

Ferguson (1986) took a different approach and added new items to the MARS to measure previously unidentified dimensions of math anxiety. His Phobos scale includes 20 MARS items selected for high loadings onto Test Anxiety and Numerical Anxiety (Richardson & Suinn, 1972) as well as ten original items, including the three examples listed in Figure 2. His new items (forming the basis of a dimension he calls "Abstraction Axiety") aimed to measure reactions to math content introduced in the middle grades, hypothesizing that this is distinct from anxiety in reaction to numerical computations associated with the lower grades.

Additionally, there have been some math anxiety scales developed totally independently of the MARS instrument. The Mathematics Anxiety Scale (MAS), developed by Betz (1978), was based on the the Math Anxiety subscale of the Fennema-Sherman Mathematics Attitudes Scale (Fennema & Sherman, 1976), and used with secondary students. And the 12-item Math Anxiety Questionnaire (MAQ) was developed by Wigfield and Meece (1988) for use with elementary school students. Kazelskis (1998) showed using a factor analysis that these two scales are different than the RMARS, indicating additional dimensions of math anxiety that the RMARS fails to measure.

Table 1 summarizes a sample of studies that find similar factor structures as the RMARS along with the studies mentioned above that add to the three factors of the RMARS.

These instruments and factor analyses show some of the possibilities of dimensions of math anxiety not contained or commonly identified in the original MARS/RMARS. This suggests that math anxiety generally can come from a number of distinct sources, and that a choosing items to test for math anxiety inherently makes a choice of what sources to consider.

In the following section, we use a deep analysis of the definition of math anxiety to argue for additional scenarios that should be included in an inventory claiming to measure it. Some of what we propose faintly echos items and dimensions from Ferguson and Bessant, but the basis of our claims for inclusion is different.

2.2 Revisiting the definition of "Math Anxiety" through a contemporary lens

As mentioned in the introduction to this Literature Review, Richardson & Suinn define math anxiety in their 1972 paper as

> a feeling of tension and anxiety that interferes with the manipulation of numbers and the *solving of mathematical problems* in a wide variety of ordinary life and *academic situations.* (p. 551, emphasis added)

Two reasons to retain this definition are that (1) it is the most commonly cited definition in the literature and (2) it is the definition that the "gold standard" MARS instrument and its descendants are based on. While most scales mention

Table 1 A selected list of factor analyses of Math Anxiety instruments. Factors are named as they were by original authors, but are noted parenthetically if similar to MARS factors from Alexander and Martray and MEARS factors from this paper.

Authors (scale abbr.)	**Year**	**Factors listed in order of strength (MARS + MEARS factors they resemble)**[5]	**# items**	**Source of items**	**Sample Population**
Rounds and Hendel	1980	Mathematics Text Anxiety (1), Numerical Anxiety (2,3)	94	MARS	350 females in math anxiety treatment program
Plake and Parker (MARS-R)	1982	Learning Mathematics Anxiety (3), Mathematics Evaluation Anxiety (1)	24	MARS	170 students in introductory statistics class
Resnick, Viehe, and Segal	1982	Evaluation Anxiety (1), Social Responsibility Anxiety (-), Arithmetic Computation Anxiety (2),	98	MARS	1045 (94%) students in a single freshman class
Ferguson	1986	Numerical Anixety (2), Mathematics Test Anxiety (1), Abstraction Anxiety (*), plus two uninterpretted factors	30	20 from MARS, 10 new (Phobos)	365 students across 18 math courses at a community college
Alexander and Martray (RMARS)	1989	Math Test Anxiety (1), Numerical Task Anxiety (2), Math Course Anxiety (3)	25	MARS	517 college students in lower level psychology
Bessant	1995	General Evaluation Anxiety (1), Everyday Numerical Anxiety (2), Passive Observation Anxiety (3), Performance Anxiety (*), Mathematics Test Anxiety (1), and Problem-Solving Anxiety (*).	80	MARS	173 undergraduates in an intro statistics course
Kazelskis	1998	RMARS items factors: Math Test anxiety (1), Numerical anxiety (2), and Course Anxiety (3). MAS items and MAQ items: Negative Affect Towards Mathematics (), Worry (), and Positive Affect Towards Mathematics ().	48	MAS (12) + MAQ (11) + RMARS (25)	323 undergradutes in college algebra

[5] (1) = Mathematics Text Anxiety, (2) = Numerical Task Anxiety, (3) = Math Course Anxiety, (*) = Explanation or Problem-Solving Anxiety, (-) = unrelated to MARS + MEARS factors

this definition only in passing, we believe that it warrants further discussion. In particular, the two phrases "solving of mathematical problems" and "a wide variety... of academic situations" emphasized above should be seen in light of their own bodies of scholarship. Understanding the meaning of these phrases in turn affects the meaning of "math anxiety" and how we should measure it.

2.2.1 A contemporary lens on Mathematical Problem Solving

Schoenfeld (1992) points out that the terms "mathematical problems" and "problem solving" have different meanings to different audiences and they have varied substantially in popular educational use over time. On a spectrum of meanings, Schoenfeld describes two conflicting interpretations of (mathematical) "problem": problems as routine exercises and problems that are problematic (pp. 337–338). "Routine exercises" are problems for which students know a method or routine to use and for which the goal of the problem is getting an answer. Numerical computations are almost always routine exercises. But more complex problems may be, too, if practiced in the context of "learn this technique, apply this technique," to many similar problems.

In contrast, "problems that are problematic" (now commonly also called *non-routine problems*) are not about applying a known method, but rather, inventing or discovering a method to solve the problem. In this conception of "problem" and "problem-solving", the *process* of finding a solution is the goal as much, if not more so, than finding the solution itself. This conception of problem-solving is what mathematicians practice daily on a more global scale—attempting to solve problems that have never been solved by anyone and do not have a pre-ordained method of solution.

It is no surprise, given our training as mathematicians, that this second conception is what we (the authors) think of as "problem-solving". We also frequently ask our students to engage in this kind of problem-solving in the courses we teach, using their knowledge base to work on new and novel problems. For these problems, as opposed to what we usually call "exercises", the method of approach is not given and generally not known from the beginning.

In addition to requisite prior knowledge (of definitions, procedures, and more overarching general strategies), solving such problems usually requires monitoring of the solving process and deciding whether to continue or discontinue a particular approach (see Schoenfeld, 1989), and a disposition toward learning and toward mathematics that allows one to make several failed attempts and continue to persevere. These necessary knowledge and behaviors are described in depth in Schoenfeld's 1985 book on *Mathematical Problem Solving*, where he sorts them into four categories: Resources (facts, procedures, etc.), Heuristics (strategies), Control (monitoring and decision-making), and Belief Systems.

Schoenfeld's work on problem-solving was mostly carried out at the college level, like the classes we teach. Another body of literature, on mathematics tasks and their cognitive demand in the K–12 setting (e.g., Doyle, 1988; Stein & Lane, 1996), presents a similar characterizations of mathematical problem solving. Specifically, the Stein & Lane definition of tasks with *High / Doing Mathematics* cognitive demand is summarized as:

> The use of complex, non-algorithmic thinking to solve a task in which there is not a predictable, well-rehearsed approach or pathway explicitly

> suggested by the task, task instructions, or a worked out example. "Doing mathematics" processes are often likened to the processes in which mathematicians engage when solving problems. (Stein & Lane, 1996, p. 58)

This description of tasks defined as "doing mathematics" aligns closely with our personal conceptions of "solving of mathematical problems," but—like "problematic problems" described above—is completely misaligned with the kinds of mathematical problems that show up in the MARS instrument and its descendants to measure math anxiety (see Figure 1 and Appendix A). We also note that although Bessant (1995) explicitly calls one of his peripheral dimensions "Problem-Solving Anxiety", the scenarios they represent—e.g., "Solving a problem such as: If $x = 12$, and $y = 4$, what is the ratio of x to y?" or "Adding up $1/5 + 2/3$ on paper."—also do not fit the definition of problem solving in the sense of problematic problems. One of the scenarios in Bessant's problem-solving dimension, "Doing a word problem in algebra," could be a problematic problem, depending on the context. But as worded, it is unclear if this item falls into this conception of problem solving or not.

It is notable that the original definition of math anxiety given by Richardson & Suinn was published in 1972, on the verge of the so-called "back-to-basics" movement in math education focused on rote routines. In this era, what the authors meant by "the solving of mathematical problems" may very well have been numerical manipulations or very simple word problems with specific strategies or routines attached to them. As our collective notion of "problem" evolves, however, so should our measurement of anxieties that stem from engaging in solving them.

2.2.2 A contemporary lens on the Mathematics Classroom

A second phrase from the definition of math anxiety that warrants discussion is "a wide variety of... academic situations". A read through the items in the RMARS (see Appendix A), the most commonly used MARS descendant, presents a very limited view of such situations. The instrument includes scenarios such as listening to a lecture, reading a textbook, and registering for courses. But the instrument fails to represent many situations that we daily ask our students to engage in and that are generally thought today to be productive classroom practices for learning mathematics, such as talking about mathematics with peers or the instructor.

In this section, we present a theoretical perspective on what the MARS items are lacking with regards to "academic situations" by looking briefly at a contemporary view of instruction as *interactions* between instructor, students, and content (Cohen, Raudenbush, & Ball, 2003). Then we turn to a practical perspective on what the MARS items are lacking by analyzing four contemporary documents describing best practices in college mathematics teaching.

Though many important academic situations take place outside the classroom—e.g., doing homework, studying, or using resources such as the textbook, tutoring center, or online videos—at its most basic, "academic situations" should include the many faces of *instruction*, that is, what happens in the classroom. In their influential perspective on instruction Cohen, Raudenbush and Ball (2003) describe: "Teaching is what teachers do, say, and think with learners, concerning content, in particular organizations and other environments, in time (p. 124)" This is com-

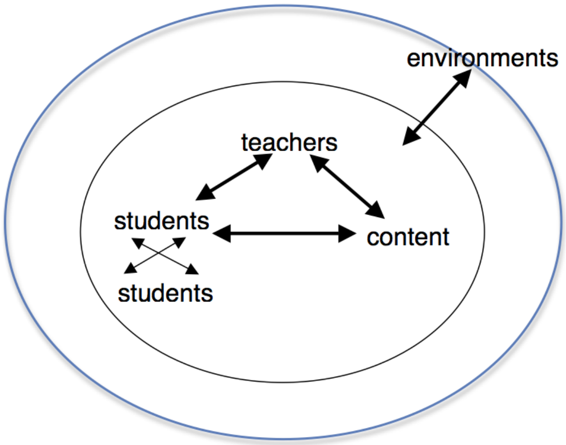

Fig. 3 "Instruction as interaction," adapted from Cohen et al., 2003.

monly referred to as the "instructional triangle" and is depicted visually in Figure 3.

The academic situations presented in the RMARS fail to represent two key interactions in this model: (1) instructor interacting with students and (2) students interacting with students. We claim that it fails to capture instructor-student interaction because none of the situations it represents are reciprocal or symmetrical—they are all unidirectional.[6]

As practical evidence for what are are generally thought today to be productive classroom practices for learning mathematics, we turn to several influential policy documents, position statements, and recommendation guides from the last decade that attend to math teaching. We take these, collectively, not as evidence of actual teaching practice, but as collective agreement to what college math teaching *should* look like. The teaching practices in these documents should, therefore, be included under the heading of "academic situations" when describing college mathematics teaching, if that is what the construct of math anxiety is supposed to measure.

The influential documents we highlight are the following:

2012 *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics*, President's Council of Advisors on Science and Technology (PCAST).
2015 *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*, Mathematical Association of America (MAA).
2016 "Active Learning in Post-Secondary Mathematics Education", College Board of Mathematical Sciences (CBMS)
2018 *Instructional Practices Guide*, Mathematical Association of America (MAA)

[6] Looking ahead at the instrument we eventually describe in §3.1: although our original items included both of these types of interactions (see Appendix B), a factor analysis grouped both types of interactions, so we focused on peer-peer interactions in the final instrument (see Table 2).

We present below some of the key findings of each report, as they relate to student engagement and interactivity, and provide a summary of pedagogical recommendations across the documents at the end.

PCAST Report

With the overall goal of increasing graduates with degrees in the Science, Technology, Engineering, and Mathematics (STEM), the PCAST report (2012) presents overarching strategies and related concrete recommendations, based on a few primary research findings. One of the first research findings they highlight is the importance of student engagement to persistence in STEM majors: "Compared with students in traditional lectures, students who play an active role in the pursuit of scientific knowledge learn more and develop more confidence in their abilities, thereby increasing their persistence in STEM majors (p. 6)." Thus, the first overarching strategy they list to improve STEM education (and retention) is: "Adopt STEM teaching strategies that emphasize student engagement (p. 8)." As a concrete recommendation, they propose: "Catalyze widespread adoption of empirically validated teaching practices (p. 16)" and highlight such evidence-based teaching practices in Table 2 on p. 17, including: Problem-Based Learning (Capon & Kuhn, 2004; Preszler, Dawe, Shuster, & Shuster, 2007) and Problem Sets in Groups (Cortright, Collins, & DiCarlo, 2005). Additional studies published too recently to be cited by the PCAST report further reinforce its message. These include Laursen's studies (with her colleagues) of Inquiry-Based Learning in college mathematics classrooms (2011; 2014) and (Kogan & Laursen, 2014), and active learning (in contrast to lecture) more generally in a meta-analysis by Freeman et al. (2014).

Common Vision

"The Common Vision project brought together leaders from five professional associations—the American Mathematical Association of Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM)—to collectively reconsider undergraduate curricula and ways to improve education in the mathematical sciences (p. 1)" In particular, this document synthesizes recommendations and findings from seven curricular guides put out by these five organizations (see pp. 11–12 in Saxe & Braddy). We highlight here the pedagogical recommendations they make in the second of their five over-arching themes in their executive summary:

> Across the guides we see a general call to move away from the use of traditional lecture as the sole instructional delivery method in undergraduate mathematics courses...Even within the traditional lecture setting, we should seek to more *actively engage* students than we have in the past. All seven guides stressed the importance of moving toward environments that incorporate multiple pedagogical approaches throughout a program. Oft-cited examples are *active learning models where students engage in activities such as reading, writing, discussion, or problem solving that promote analysis, synthesis, and evaluation of class content.* Cooperative learning, problem-based learning, and the use of case studies and simulations are also approaches that actively engage students in the learning process. (Saxe & Braddy, 2015, p. 19; emphasis added)

CBMS Statement
In 2016, the College Board of the Mathematical Sciences put out a short position statement in support of Active Learning. In the short statement, the entire following sentence is bolded and italicized:

> We call on institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers, and funding agencies to invest time and resources to ensure that effective *active learning* is incorporated into post-secondary mathematics classrooms. (Conference Board of the Mathematical Sciences, 2016, p. 1; emphasis added)

MAA Instructional Practices Guide
The Instructional Practices Guide represents a professional organization's recommendations on teaching and is, to a large extent, based on evidence from the research literature. This document opens with an extensive chapter on strategies and practices for fostering student engagement in the classroom. Nine, somewhat overlapping, strategies are highlighted and described:

CP.1.1. Building a classroom community
CP.1.2. Wait time
CP.1.3. Responding to student contributions in the classroom
CP.1.4. One-minute paper or exit tickets
CP.1.5. Collaborative learning strategies
CP.1.6. Just-in-time teaching (JiTT)
CP.1.7. Developing persistence in problem solving
CP.1.8. Inquiry-based teaching and learning strategies
CP.1.9. Peer instruction and technology

Several meta themes cross-cut many of the described practices. Two cross-cutting themes we highlight below are notable for how pervasive they are across the practices highlighted.

1. **Eliciting and responding to student-thinking (right or wrong).** E.g., "To encourage student responses and participation, it is important to recognize the value of students offering both correct and incorrect responses. (p. 15)". In some cases, such as Exit Tickets (CP.1.4) and Just-in-time teaching (CP.1.6), this formative assessment is private. But in many of the practices (CP.1.2, CP.1.3, CP.1.5, CP.1.8, CP.1.9[7]), student thinking is elicited *and* made public in the classroom.
2. **Student Collaboration**. E.g., "Given that students are working on problems that are designed to be engaging, it often means that these problems are also more difficult than standard problems and require collaborating with peers... These collaborations facilitate learning to form logical arguments and as a result students are able to tackle more difficult problems. (p. 35)" This overlaps with student thinking being public, but in a particular mode where students are responding to each other directly, usually in pairs or small groups. This overarching practice shows up explicitly in the descriptions of CP.1.5 and CP.1.8, and shows up in passing in the descriptions of CP.1.1, CP.1.3, CP.1.7, and CP.1.9.

[7] In CP.1.9. students' answers to questions via device/clicker are anonymous—so private—but the ensuing peer-to-peer discussion is still public.

Looking across these four documents, we see that current, research-based recommendations for college mathematics teaching[8], as endorsed by the prevailing, relevant professional societies, include: eliciting and using student thinking, peer-to-peer collaboration, and students actively working on mathematics content during class. None of these central instructional practices or classroom situations are captured in the MARS/RMARS items, but they are increasingly important to any representation of mathematical academic situations.

2.2.3 Math Anxiety in an interactive classroom

Not only do the current math anxiety instruments fail to capture academic situations reflective of current pedagogical recommendations, but not much has been studied about math anxiety in such classrooms. A search of the ERIC database for the term "mathematics anxiety" currently yields 1861 results[9]. However, adding the keyword "active learning"—the appropriate term in the ERIC thesaurus for our setting—narrows these results to fourteen. Of those fourteen, there are no peer-reviewed journal articles relevant to math anxiety in an interactive, post-secondary, mathematics classroom. A search of the JSTOR database lead to similar findings: a full text search of articles in the database for

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(("mathematics anxiety") AND ("active learning" OR "Inquiry Based
            Learning" OR "Inquiry-Based Learning"))
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finds fourteen articles. However, none were relevant to our setting of interest.

While these searches are by no means exhaustive, they are indicative of the dearth of research in the area of math anxiety pertaining to post-secondary interactive mathematics classrooms. In particular, we have found no research on (1) how an interactive classroom might affect students' math anxiety, nor (2) what dimensions of math anxiety are relevant in an interactive classroom.

While these are not the questions this present study aims to answer, our new items are, by design, poised to better study these kinds of classroom settings than the RMARS alone.

2.3 Math anxiety and elementary teachers

We developed our survey in the context of our work with pre-service elementary teachers (PSETs), for whom classroom mathematics is not only the context in which they are trained but also the context in which they will work. Historically, elementary school teachers are a much-studied group when it comes to math anxiety. This is likely both because they exhibit the largest amounts of math anxiety among university students grouped by major (Hembree, 1990) and because they will be in a position to either propagate or alleviate math anxiety by how they teach mathematics. An alarming finding of Beilock, Gunderson, Ramirez, & Levine (2010) is that female elementary teachers' math anxiety negatively affects the mathematical skills of their female students, by enforcing gendered ideas about who is good at math.

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[8] Note that these guides also attend to *content* recommendations to some extent, but as we delve into the meaning of "problem solving" in the previous section, §2.2.1, we avoid further expansion on that subject here.

[9] As of September 2018.

Various studies show that teachers' math anxiety is related to, but not the same as, their anxiety about teaching mathematics. Peker (2009) defines mathematics teaching anxiety (MTA) as "pre- and in-service teachers feelings of tension and anxiety that occurs (sic) during teaching mathematical concepts, theories, and formulas or during problem solving." Brown et al. (2011) found that, while Math Anxiety (MA) directly correlated with Math Teaching Anxiety (MTA) for a majority of elementary PSETs, there are a significant number of PSETs that either have MA without MTA or MTA without MA (about 36% of students in their study, combined). Using a different set of instruments to measure MA and MTA, Peker and Erteken (2011) found a "positive, moderate relationship" between the two. McAnallen wrote a survey to measure anxiety and self-efficacy in mathematics and mathematics teaching (McAnallen, 2010) that yielded two factors: one related to mathematics teaching, and one related to mathematics outside a teaching context. In a survey of 691 in-service elementary teachers, she found a significant ($p < 0.001$) correlation of 0.63 between the two factors.

Taken together, these studies suggest that a teacher's math anxieties are often related to their math teaching anxieties, and that these have real influence on the educational experiences of their students. Thus, it is important to have a measure of math anxiety that aligns well with the style of math we ask teachers to engage with in the classroom.

What kind of mathematics are we asking teachers to engage with in the classroom? For evidence of this, we turn to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), a contemporary standards document adopted by forty-one states and four US territories. In addition to specific content standards, the CCSS are framed by over-arching *math practices*. These describe what it means to "do mathematics" in a K-12 classroom.[10] By looking at these math practices that teachers are expected to engage their students in, we can see a close connection between this work and the work of teachers doing mathematics themselves (as described in §2.2).

The CCSS Standards for Mathematical Practice consists of eight practices:

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.

We want to point out from this list of practices is that MP1 and MP3, in particular, closely mirror the attributes of "solving mathematical problems" in a "variety

[10] While the idea of math practices has gained traction through documents like the Common Core State Standards (CCSS) (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), we note that math practices are much more general than the eight listed in this document. The ten process standards in the National Council of Teachers of Mathematics Principles and Standards (2000) can be seen as kinds of math practices, as can the competencies in *Adding it Up* (National Research Council and Mathematics Learning Study Committee and others, 2001). And Bass has described math practices as practiced by mathematicians (2011).

of mathematical situations" (described in §2.2) that we are proposing to more robustly measure. That is, what we are proposing to more robustly measure with regards to the definition of math anxiety, has a natural parallel in the task of teaching, as outlined by the current CCSS for mathematics. While we already described known connections between MA and MTA above, this suggests that our new measures are potentially even more closely connected to math teaching.

3 Instrument Development and Data

3.1 Instrument History and Development

As part of an earlier study of students' performance on oral assessments (White & Visscher, 2015), we used the RMARS to measure math anxiety but added seven supplemental items to better reflect the mathematical situations students would encounter in our interactive classroom. To craft these items initially, we considered scenarios that incorporated both math practices enacted by students in our classrooms (e.g., CCSS MP1: "Make sense of problems and persevere in solving them" and CCSS MP3: "Construct viable arguments and critique the reasoning of others") and pedagogical practices we used as instructors in interactive mathematics classrooms (e.g., create opportunities for students to struggle with novel problems, ask groups of students to discuss results and come to a consensus, ask students to present their solution to the class). We focused on the kinds of practices and scenarios that in our experience made some students seem especially anxious or nervous, but that weren't represented well in the RMARS. See Table 7 in Appendix B for these seven pilot items.

An exploratory factor analysis of the resulting data ($N = 44$) grouped the new items into two factors that we found highly interpretable and subsequently named Problem Solving Anxiety and Explanation Anxiety (see Table 8 in Appendix B). Our study showed some connection between these two factors and student performance on different styles of assessment[11] and we decided that these anxieties should have a more systematic measurement (White & Visscher, 2015).

The items designed to measure Problem Solving Anxiety and Explanation Anxiety in the present MEARS instrument were developed in a more systematic manner. We started by defining the constructs we wished to measure and identifying several sub-constructs that we thought could independently contribute to either Problem Solving Anxiety or Explanation Anxiety:

Problem Solving Anxiety is a feeling of anxiety in response to encountering

- a conceptually difficult or novel math problem (for example, a problem in unfamiliar terrain, or that one has trouble making progress on), or
- a lengthy problem or set of problems.

[11] As a quick summary of the connections found: students with higher Problem Solving Anxiety seemed disadvantaged on traditional, written assessments compared to individual oral assessments, but students with higher Explanation Anxiety had the reverse disadvantage.

Explanation Anxiety is a feeling of anxiety in anticipation of or response to explaining one's mathematical ideas to others.

A person may exhibit this anxiety differently in different types of situations:

- *Explanation Anxiety with External Doubt* is explanation anxiety in the presence of someone else expressing doubt about the ideas;
- *Explanation Anxiety with External Validation* is explanation anxiety in the presence of someone else sanctioning the ideas as correct;
- *Explanation Anxiety with Internal Doubt* is explanation anxiety in the presence of doubting the correctness of the ideas oneself.

These constructs, which grew out of the factor analysis of our experience-based items in our initial study, correspond to two major facets of "anxiety that interferes with ... solving mathematical problems... in a wide variety of... academic situations" that we discuss in §2.2 as missing from the MARS instruments and its derivatives.

For each sub-construct, we wrote three items that we thought would each directly measure anxiety in the described situation. This was intended to minimize undesired effects due to wording or alternate interpretations of the items. After running the study, we decided that one item written for "a lengthy problem or set of problems" was better aligned with "a conceptually difficult or novel math problem"; the data backed up this assertion. Hence, one sub-construct for Problem Solving Anxiety has four items while the other has two. We also consulted with a psychometrician and edited the item statements to improve the likelihood that they would be consistently interpreted. An additional step that would have improved our items even further would have been several in-person discussions with students taking the survey to make sure the items were coming across as intended. We discuss this further in a general section on the limitations of our methods (see §3.4).

3.2 Participants

We recruited participants through a professional network of mathematicians involved in teacher education. Among instructors that responded, some gave the survey during class time and some emailed the link to their students. After removing incomplete responses and responses from students who did not identify as future elementary teachers, the survey sample consisted of 132 primarily undergraduate pre-service elementary teachers.[12] Demographic data for this sample is reported visually in Figure 4.

[12] We had 162 total responses to the survey. Thirteen of those respondents answered "No" to being a pre-service teacher and sixteen did not answer that question. After removing those 29 responses, there was one additional response for which the respondent had clearly filled out the survey without actually reading any of the questions—they entered the same option for every single question; we removed that response as well, leaving us with 132 responses to analyze.

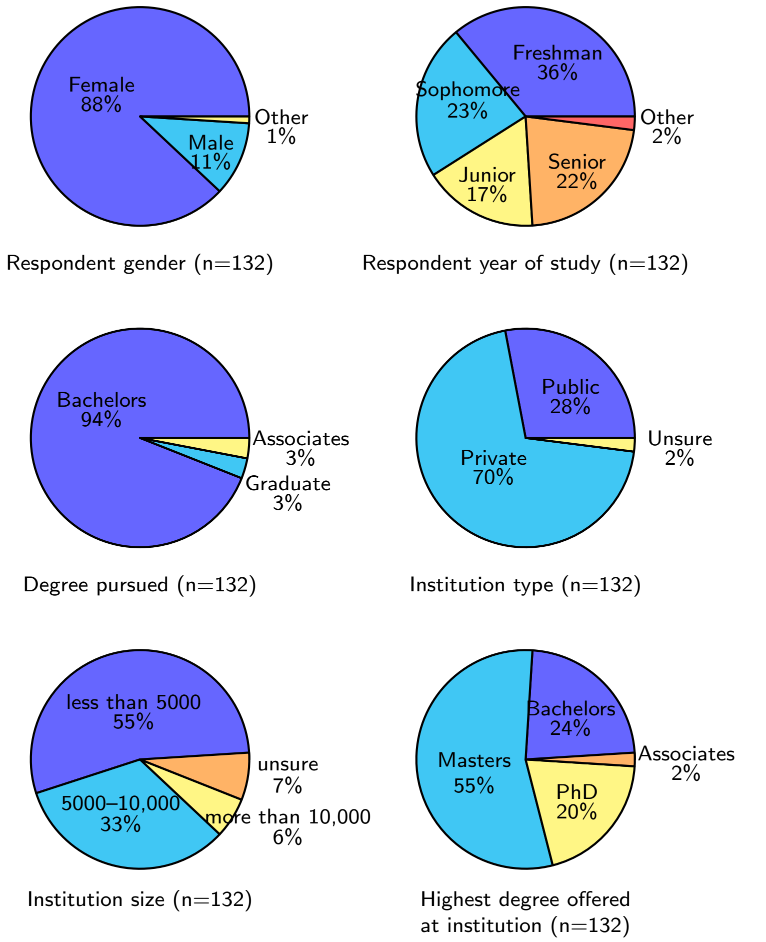

Fig. 4 Summary statistics for the sample used in this study (132 undergraduate pre-service elementary teachers).

3.3 Data and analytical methods

We collected data via an online survey on the Qualtrics platform. The central component of the survey was our new instrument we were validating: the Mathematics Engagement Anxiety Rating Scale (MEARS), consisting of 15 newly developed items. This was supplemented by the 25 items of the RMARS. In order to test for general anxiety, participants also completed the State-Trait Anxiety Inventory (STAI), a standard 20 item survey for general anxiety. Finally, because we hypothesized that our items measuring Problem Solving Anxiety might be related to a fixed mindset (Dweck, 2007), participants also responded to four mindset questions taken from the work of Carol Dweck. We discuss this hypothesized connection and its relevance in more detail in §5.1.

We analyzed our data to: (1) produce and verify a factor structure for our MEARS items, (2) check that these factors are distinct from factors in RMARS as well as general anxiety, (3) measure the internal consistency of our factors and the instrument in general and (4) validate the instrument (i.e., make sure that our items measure what we say they do). Here, we provide a brief description of the tests we used. The results of these tests are presented in Section 4.

An exploratory factor analysis (EFA) creates a specified number of factors and generates item loadings to the factors that best fit the data set. This is independent of any theoretical model for the instrument. An EFA can be used to determine the appropriate number of factors as well as test for unexpected patterns in the data. There are different types of rotation methods used in EFA for fitting the data; the major difference is between orthogonal and oblique rotation methods. Since the factors we wish to measure are very likely correlated, we use promax rotation—an oblique rotation method.

Confirmatory factor analysis (CFA) takes a specified theoretical model and reports on how the data fits into the model. Fit indices give an indication of how well the model describes the data; a CFA also reports loadings of items onto specified factors. The models we test are based on our design of the questions but also informed by the results of running a few EFAs. In order to determine if the theoretical model is a good fit for our data, we use the AIC model fit index because it penalizes the complexity of the model (so that simpler models are preferred).

Cronbach's alpha is a standard test for internal consistency. A scale or subscale is internally consistent if its items measure the same (or similar) things. This is measured by comparing the within-subject variability of a scale or subscale against the between-subject variability. The range of the alpha score is between negative infinity and one: negative alpha scores indicate that there is greater within-subject variability than between-subject variability, while scores above 0.9 are generally considered to indicate strong internal consistency. Very high alpha scores (above 0.95) can indicate that items are redundant.

Pearson's correlation coefficient indicates the degree to which two variables are linearly correlated. The range is between -1 and 1, with 1 indicating perfect correlation, negative values indicating anti-correlation, and 0 indicating no correlation. This test is useful to show that two variables *are correlated*—that is, it tests against a null hypothesis that they are not correlated. We check that various factors and constructs are correlated, though this result is not surprising. In order to check that a factor is distinct from other constructs, we run EFAs to see how items are clumped into factors; if an EFA separates the factors as expected, then we consider them distinct constructs.

3.4 Limitations

We note several limitations in our item development, data collection, and sample.

The item writing and editing was done collaboratively between the two authors, incorporating our two different perspectives. A psychmetrician was also consulted to provide a sounding board for the meaning and interpretability of our items. There was, however, room for further external validation of our items. Foremost, a read-through with a survey taker (or more than one) would have more conclusively ensured that items were being interpreted as intended. (The consultation with the psychometrician fulfilled this to some degree.) Another step would have been to consult other mathematics instructors for feedback; this step could still be taken and is a process would likely expand greatly on the overall the number of items. However, we do not propose that the items we've written complete the picture of the dimensionality of math anxiety. We discuss other dimensions one might measure in our discussion in §5.3.

Despite the lack of cognitive interviews by survey takers, we claim that our new items are valid based on the balance of the three perspectives that shaped them, and the results of the factor analysis reported in §4 that the intended constructs did, indeed, show up as separate factors. The fact that our list of items is not yet complete is not an argument against the validity of the additional dimensions of math anxiety we've so far identified.

The authors intended for the question order to be randomized when subjects took the survey, but, due to a technical glitch with the Qualtrics platform, all participants had received the questions in the same order. The robustness of the quantitative results and the qualitative design of the items, however, makes us confident that the factor analysis would produce similar results with randomization. We have evidence of this also in the fact that items from the original RMARS were pulled into the new factors in a few cases (see Table 9), even though they weren't listed in a similar place in the survey.

Finally, our sample has at least two limitations: (1) while our sample size ($N = 132$) is large enough for validating a survey of this length (15 items), a larger sample would have allowed for more reliable factor analysis of the 40-item combined instrument (RMARS + MEARS; see §4.2); (2) we intentionally restricted our sample to pre-service elementary teachers, because we were personally most interested in their math anxiety levels and what effects that might have on their future teaching practice. However, this survey and these subscales are likely relevant to more general populations, too, and should be validated with a more diverse sample.

4 Results

This section is organized into three parts: (1) analyses of MEARS items in isolation, (2) analyses of the RMARS and MEARS items together, and (3) correlations between the MEARS factors and other instruments. Brief background on the statistical methods we use can be found in §3.3.

4.1 Analyses of MEARS items.

As discussed in §3.1, the MEARS items were written with two factors in mind: anxiety around solving novel or lengthy problems (Problem-Solving Anxiety), and anxiety around explaining mathematics to a peer or instructor (Explanation Anxiety).

An exploratory factor analysis (EFA) on the 15 MEARS items showed that a three factor model is the best fit for our data[13]. These three factors reported by the EFA have very natural interpretations: one factor is made up of all six Problem-Solving items, while the nine items written for Explanation Anxiety splits into the

[13] "Best fit" was determined using the criteria that additional factors do not significantly increase the cumulative amount of variation explained by the model.

remaining two factors—those involving external validation or doubt (6 items), and those involving internal doubt (3 items). We further discuss these factors in §5.1. Specific item loadings for the three-factor EFA are reported in Table 2.

Table 2 Factors and item loadings for an EFA of the 15 MEARS items.

I. Problem Solving Anxiety (PS-Anx)	
Items pertaining to difficulty of problem.	
1. Working on a math homework problem and not making any progress for 5 minutes.	.92
2. Being asked to solve a math problem when you are not sure which formulas to use.	.84
3. Being given a math problem that does not look like any problem you have seen before.	.80
4. Working on a math problem for which you are not sure where to start.	.79
Items pertaining to length of problem set.	
5. Being assigned an extra long math homework set.	.89
6. Beginning to work on a multi-page math worksheet.	.89
II. Explanation Anxiety (E-Anx)	
Items involving external doubt	
7. Being asked to further justify why your mathematical solution is correct to a classmate who is not yet convinced.	.98
8. Having to convince a classmate that your different way of solving a math problem is equally valid.	.90
9. Continuing to explain your mathematical solution, even though a classmate doubts it is correct.	.75
Items involving external validation	
10. Being asked by a classmate to go through your correct solution more slowly.	.83
11. Describing to a small group of classmates how you went about a homework problem on which you received a perfect score.	.69
12. After reaching an “aha!” moment on a problem on your math worksheet, being asked to explain your solution to a small group of classmates.	.64
III. Explanation with Internal Doubt Anxiety (EID-Anx)	
Items concerning mathematical explanation with internal doubt	
13. Sharing your solution with a small group of classmates when you are not sure it is correct.	1.07
14. Explaining your attempt at a math problem to a classmate, even though you are not very convinced that it is right.	.83
15. When you are partway through figuring out a math problem, being asked to share your thinking with a classmate.	.58

Eigenvalues are 4.45 for Factor I, 3.99 for Factor II, and 2.27 for Factor III.

We subsequently ran a confirmatory factor analysis (CFA) for the following models of the MEARS items:

- One single factor (15 items);

– Two factors (as survey designed): PS-Anx (6 items) and original Explanation Anxiety (9 items);
– Three factors (from EFA): PS-Anx (6 items), E-Anx (6 items), and EID-Anx (3 items).

There are many indices that report on how well a model fits the data; the AIC is a model fit index that penalizes complexity of the model. Among these three models, the lowest AIC is for the three-factor model. We note that running a CFA for a five-factor model (using the five most precise subscales as the survey was written, cf. §3.1) lowers the AIC further. We do not pursue this model because these five factors are part of the sub-factor structure as we designed the instrument and it becomes a bit unwieldy to consider five factors independently.

The EFA and CFA results indicate that the items are optimally sorted into three factors: Problem Solving Anxiety (PS-Anx), Explanation Anxiety (E-Anx), and Explanation with Internal Doubt Anxiety (EID-Anx). The EFAs indicate that the cumulative variation explained by n factors plateaus at $n = 3$, and the AIC model fit index for CFAs with different models shows that this model best explains the data among the readily interpretable and useful theoretical models of the instrument.

Finally, as a standard test of internal consistency, we computed the Cronbach's alpha scores for the three factors found in the above exploratory and confirmatory factor analyses. This yields $\alpha = 0.94$ for Problem Solving Anxiety, $\alpha = 0.92$ for Explanation Anxiety, and $\alpha = 0.91$ for Explanation with Internal Doubt Anxiety. These scores indicate that there is much more variation between subjects than item-to-item within these subscales and that the constructs are internally consistent. As a whole, the survey has $\alpha = 0.94$, and so can be considered internally consistent. We note that scores larger than 0.95 are often considered an indication that questions are redundant and almost always produce the same response from subjects; our data analysis shows that this is not the case in our instrument.

Strength of MEARS Factors. In order to compare the amount of anxiety different factors cause in our study population, we compute the average score for the items that comprise each factor; this is called the factor score. Averages and standard deviations of the factor scores from our data for each of the three MEARS factors are reported in Table 3. For comparison, the meta-analysis (Hembree, 1990) reports a mean score of 2.24 (on the same scale) for Elementary education majors on the 98-item MARS. Our study population had a mean factor score of 3.05 for Test Anxiety, 1.56 for Numerical Anxiety, and 1.86 for Course Anxiety. The mean score for the entire 25-item RMARS instrument was 2.52.

Table 3 Averages of per-item scores for the three MEARS factors. The range is from 1 to 5, with higher numbers indicating more anxiety.

	mean	sd	median
Problem Solving Anxiety	3.17	0.99	3.08
Explanation Anxiety	1.98	0.89	1.83
Expl with Internal Doubt	2.54	1.10	2.33

4.2 Analysis of RMARS+MEARS

We analyzed the total 40-item inventory (RMARS + MEARS) in order to determine if our new items were measuring something distinct from the RMARS constructs of Math Test Anxiety, Numerical Anxiety, and Math Course Anxiety.

Although we do not have enough statistical power to conclude much from a factor analysis on all 40 items together, the findings nonetheless have evident descriptive value. An EFA for the 40 items of RMARS+MEARS with five factors redistributes some of the RMARS items among the new factors in particularly interpretable ways. In particular, the first factor[14] in this EFA is Problem Solving Anxiety, and it pulls in three items from RMARS that make sense given our definition of PS-Anx (see §3.1):

6. Being given homework assignments of many difficult problems that are due the next class meeting.
11. Picking up a math textbook to begin a difficult reading assignment.
13. Opening a math or stat book and seeing a page full of problems.

Note that Alexander & Martray (1989) report these three items as belonging to the Math Test Anxiety factor, but that they don't conceptually align well with that factor. So it's especially unsurprising that they would load with one of our factors instead. For the reports that follow, we group these three items into our Problem Solving Anxiety factor and remove them from the Math Test Anxiety factor.

The second factor is Math Test Anxiety (with fewer items), followed by Numerical Anxiety, Explanation Anxiety (6 items, see Table 2), and Explanation with Internal Doubt Anxiety (3 items, see Table 2). The Math Course Anxiety factor from RMARS does not appear in this analysis; items from the Alexander and Martray Math Course Anxiety factor do not load significantly onto any of the factors. The redistributed items are reported in Appendix C.

Table 4 Factors and the variation they explain of a 5-factor EFA on all 40 items (RMARS + MEARS).

Factor	Variation explained
Problem Solving Anxiety†	19%
Math Test Anxiety†	18%
Numerical Anxiety	14%
Explanation Anxiety	11%
Expl with Internal Doubt	5%

† Problem Solving Anxiety in this table refers to the 9-item factor that includes the three additional RMARS items pulled in, rather than the designed 6-item construct. Math Test Anxiety in this table similarly has fewer items than the RMARS factor of the same name.

To further investigate the distinctness of our factors from the RMARS factors, we computed correlations between the three factors of the MEARS found above in §4.1 and the original three factors from the RMARS. Unsurprisingly, al the factors are positively correlated. The highest correlation is between Problem-Solving

[14] Factors are reported in order of how much of the variation in scores they explain; see Table 4

Anxiety in the MEARS and the Test Anxiety in the RMARS, at 0.73. However, if we remove the items from Test Anxiety that loaded with Problem-Solving Anxiety in the 40-item EFA, then this correlation goes down to 0.666. All the correlations are reported in Table 5.

Table 5 Correlations between MEARS factors (as reported in §4.1) and original RMARS factors.

	Prob Solv	Explanation	Expl-doubt	Test	Numerical	Course
Prob Solv	1.000					
Explanation	0.571	1.000				
Expl-doubt	0.593	0.645	1.000			
Test	†0.730	0.534	0.551	1.000		
Numerical	0.411	0.553	0.482	0.474	1.000	
Course	0.577	0.590	0.543	0.616	0.672	1.000

† If we take out the three items from the Math Test Anxiety that loaded with Problem Solving Anxiety (see Table 9), the correlation of probsolv and test is 0.666.

Putting all this together, the fact that our MEARS items showed up in distinct factors from the RAMRS items (with the exception of a few items that conceptually made sense) in the EFA, and the fact that the correlations between the factors are only moderate (between 0.3 and 0.7) support the claim that our new items are distinct from existing dimensions and constructs in the RMARS.

4.3 Relationships between MEARS factors and other instruments.

To ensure that our new instrument was not simply measuring general anxiety, we ran an EFA (with promax rotation) on the 15 MEARS items with the 20 STAI items. An analysis with two factors yields a first factor with eigenvalue 8.28 consisting of exactly the 15 MEARS items, and a second factor with eigenvalue 7.67 consisting of exactly the 20 STAI items. These two factors cumulatively explain 46% of the variation in responses. An analysis with four factors retains the factor composed of the 20 STAI items (eigenvalue 7.31) and breaks up the 15 MEARS items into the three factors that we found previously: Problem Solving Anxiety (eigenvalue 4.44), Explanation Anxiety (eigenvalue 4.05), and Explanation with Internal Doubt Anxiety (eigenvalue 2.49). These four factors cumulatively explain 52% of the variation in responses. Other numbers of factors display similar results, keeping simple items from MEARS separate from simple items from STAI, and creating more complex items as the number of factors goes up. Taken together, these analyses suggest that our new items are not simply measuring general anxiety.

While the analyses described above suggest that the MEARS items measure something distinct from general anxiety, they are, unsurprisingly, positively correlated with STAI scores. To test this, we created a Problem Solving Anxiety factor score by taking the average of the six items associated with that factor; similarly for Explanation Anxiety and Explanation with Internal Doubt Anxiety. The correlations with STAI score (averaged with appropriate items inverted) are 0.288 (PS-Anx), 0.348 (E-Anx), and 0.386 (EID-Anx). These are statistically significant at the $p = 0.001$ level. We note that these relatively weak correlations agree with the result found above that these factors are distinct from general anxiety.

We initially hypothesized a relationship between the MEARS factors—specifically Problem-Solving Anxiety—and Mindset questions (see more discussion of this in 5.1). In particular, we thought there might be a positive, significant correlation between a student having a fixed mindset and having high Problem-Solving Anxiety. We found that all three factors of the MEARS were positively, significantly correlated with having a fixed mindset. From our data, the correlations with fixed mindset are 0.315 (PS-Anx), 0.449 (E-Anx), and 0.273 (EID-Anx). All of these are statistically significant at the $p = .01$ level.

Running factor analyses (2-factors and 4-factors) on the MEARS items plus Mindset items yielded factors that completely separated the Mindset items and MEARS factors. Together, this indicates that the MEARS factors are measuring something related to, but notably distinct from, having a fixed mindset.

5 Discussion

The results in the previous section show that pre-service elementary teachers have math anxieties that are distinct from those measured by the RMARS. Two of these we have labeled Problem Solving Anxiety and Explanation Anxiety. In this section, we discuss (1) possible causes, effects, and ways to address these anxieties, (2) their applicability to different populations, including PSETs, and (3) the potential for measuring other math anxieties in a similar vein.

5.1 Digging Deeper into Problem Solving and Explanation Anxieties

Problem Solving Anxiety has a particularly large effect, not only in the amount of anxiety it produces on average (see Table 3) but also how it explains more of the variation of scores than any of the standard factors in RMARS (see Table 4).

When we reflect on what is known about students' epistemological beliefs about mathematics, it's not surprising that being faced with problems for which the method of approach is unknown would cause anxiety. In particular, in an analysis of 33 articles from 1980 to 2004, Muis (2004) found several common non-availing beliefs in students (across grade levels) which included: students believe they are unable to construct new mathematical knowledge and solve problems on their own, that mathematics is about applying known procedures from the teacher or from the book, and that when a math problem cannot be solved within 5–10 minutes something is either wrong with the problem or with the student. When a student holding these beliefs is confronted with a problem that cannot be solved quickly with a known procedure, there is a conflict between the task and the students' beliefs. We believe this is an underlying mechanism to the Problem Solving Anxiety we have identified.

However, this explanation is not perfect. Recall that our items were initially designed to measure two separate kinds of anxiety associated with solving problems: one for particularly novel problems (Q1–Q4, Table 2)—that conflict with the non-availing beliefs about mathematics that many students hold—and one for a lengthy *list* of problems (Q5–Q6, Table 2). However, these did not show up as separate factors in our analysis. Since we didn't have enough statistical power to look for a large number of factors, this could be the reason we didn't see any distinction here. Or, it could be that a lengthy list of routine problems still brings on a sense of panic similar to a problem for which the method of approach is unknown.

Another reason that our problem-solving scenarios (especially Q1–Q4, Table 2) might cause pronounced anxiety is that students might see their not knowing how to proceed as a measurement of their intelligence. This would especially be the case for students with a fixed mindset (Dweck, 2007). In that case, being stuck or unable to do a problem might seem to mean more than simply being stuck, it might feel like an indicator of their overall intelligence, which would, understandably, cause anxiety. This line of reasoning—that students with a fixed mindset might have increased anxiety in the face of a problem they don't know how to approach—led us to include a short mindset inventory in our initial data collection. As described in §4.3, we did find that fixed mindset and PS-Anx were correlated, but also that they were distinct constructs.

The factor analysis of the items relating to Explanation Anxiety held some surprises for us. Namely, we were surprised that the six items Q7–Q12 (see Table 2) showed up as a single factor, even though three of the items described scenarios with external doubt and three described scenarios with external validation. What did fall out as a separate factor were scenarios describing the presence of internal doubt. That is, it seems that doubting your own solution you are explaining is notably distinct from having someone else doubt a solution you are explaining. We had predicted that doubt from an external source would carry more weight—and be more similar to internal doubt—than what the factor analysis showed. In retrospect, knowing that your own solution is wrong, but having to explain it anyways, does seem notably different than having someone else question something you can at least reasonably stand behind.

5.2 Survey utility: pre-service elementary teachers and general population

The MEARS items were developed for use and tested on pre-service elementary teachers, but we also see use for this survey in broader populations. In this section, we explore the special relevance of problem-solving anxiety and explanation anxiety to PSETs and we describe other potential uses for the instrument.

A question of persistent interest in teacher education research is: "what influences teachers' instructional decisions?" A teacher who experienced anxiety as a student in an interactive classroom might hesitate using interactive techniques in their own classroom, either to avoid encountering those situations that make they themselves anxious or to shield their students from potentially anxiety-inducing activities. A hypothetical example: a teacher with Problem-Solving Anxiety may focus only on procedures in her curriculum and avoid all of the open-ended problems in the textbook, either because she is anxious about encountering unexpected or unfamiliar student work, or because she remembers her own experience with such problems as as student and doesn't want her students to have the same negative experiences. Similarly, if a teacher has Explanation Anxiety, she may teach math in a more authoritarian and "here are the correct procedures" kind of way to spare her students having to explain their thinking.

For the reasons described above, we think any study measuring math anxiety in pre-service or in-service teachers to better understand their teaching decisions, classroom climate, or other instructional factors should consider the utility of augmenting the MARS with our new MEARS items and the dimensions they represent.

Looking beyond the specific population of pre-service teachers, the math anxiety factors in MEARS are more broadly relevant to anyone who encounters a need to engage in mathematical problem solving or explain their mathematical ideas. These activities are hallmarks of many active learning classrooms. Better understanding student experiences in these classrooms—for example their anxieties around certain classroom practices, as measured by our instrument—could give both practitioners and researchers insight to better fine-tune the affective dimension of teaching with these kinds of active and interactive approaches.

Individual instructors may find information from this instrument useful in their particular classrooms. To optimize learning for their students, individual instructors should be aware of how their students *feel*, not just what they know. For example, it's easy to imagine that a student whose previous mathematics classroom expectations included reproducing a litany of procedures, sitting quietly while a teacher lectures, or working individually on all assignments, might feel overwhelmed or anxious entering a mathematics classroom where expectations include explaining their mathematical thinking, working on novel problems, and questioning the mathematical thinking of their peers. While understanding the affective dimension of teaching relies on empathy and interpersonal skills, an instrument like the MEARS could give individual practitioners, programs, or departments another tool for understanding the supports students need to engage with these interactive classroom practices successfully.

Researchers studying larger scale samples of secondary or college active-learning math classrooms might also be interested in systematically measuring our expanded dimensions of math anxiety. One might ask how a students' problem-solving or explanation anxiety, for example, affects their ability to participate in such a classroom. This may shed light on more equitable or less equitable ways of enacting or supporting active learning techniques in the math classroom.

5.3 Even more dimensions of math anxiety

We developed the MEARS to measure students' anxiety around solving (challenging) mathematical problems in a classroom that includes significant peer-peer interactions. For this purpose, we focused on two math practices that—problem-solving and explaining one's thinking— that (1) are relevant to the definition of math anxiety (see §2.2), (2) are cornerstones of our problem-based, active-learning classrooms, and (3) seem to cause anxiety among students in our past experience. However, there are many other mathematical practices one might consider measuring anxiety around. For such measurements, further additional survey items would be needed.

Often-referenced lists of mathematical practices also have been published for the K-12 setting by the National Council of Teachers of Mathematics (2000), the National Research Council (2001), and in the Common Core State Standards (2010). Our items are most relevant to (1) the CCSS Practice Standard 1: "Make sense of problems and perservere in solving them" or the NCTM standard "Problem solving", and (2) the CCSS MP3: "Construct viable arguments and critique the reasoning of others" or the NCTM standard "Communication". In addition to these, we imagine that standards such as the CCSS MP2: "Reason abstractly and quantitatively" or the NCTM standard "Reasoning and Proof" may cause anxiety among students.

There are also math practices that are not necessarily represented in the K-12 publications listed above. Hyman Bass provides a list from the perspective of a mathematician with a view toward mathematics instruction of eleven math practices: questioning, exploring, representing, structure seeking, consulting, connecting, proof seeking, being opportunistic, proving, analyzing/evaluating proofs, and exercising judgment and taste (Bass, 2011). We can imagine, for instance, a situation in which students are asked to exercise judgment and taste while doing mathematics producing anxiety among some students who believe that the discipline of mathematics has one correct way of solving problems.

6 Conclusion

A contemporary conception of "solving mathematical problems" and "academic situations" necessitates that we reevaluate the way we measure "math anxiety". Our overall claim in this study is that there is more to math than crunching numbers and taking high stakes tests, and that to understand and address anxieties around mathematics, these other attributes of doing mathematics must be considered. When we wrote additional Likert-scale items to supplement the RMARS and capture aspects of doing mathematics that were noticeably missing, we found that, indeed, our new items were measuring something distinct from existing factors. This is important because it means that the current most widely-used instrument—RMARS—is missing real, distinct components of math anxiety. If we are more comprehensive in what we measure, we can be better equipped to understand and address anxieties around doing mathematics.

The connection to teaching mathematics is potentially especially potent. While its already been shown that math anxiety in female teachers (using the RMARS alone) has startling negatives consequences for their female students achievement (Beilock et al., 2010), we think there are even deeper connections to explore here between a teachers anxieties and teaching decisions they make. We hope this new supplemental instrument will help support such exploration.

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A RMARS items

Math test anxiety
1. Studying for a math test.
2. Taking math section of the college entrance exam.
3. Taking an exam (quiz) in a math course.
4. Taking an exam (final) in a math course.
5. Picking up math textbook to begin working on a homework assignment.
6. Being given homework assignments of many difficult problems that are due the next class meeting.
7. Thinking about an upcoming math test 1 week before.
8. Thinking about an upcoming math test 1 day before.
9. Thinking about an upcoming math test 1 hour before.
10. Realizing you have to take a certain number of math classes to fulfill requirements.
11. Picking up math textbook to begin a difficult reading assignment.
12. Receiving your final math grade in the mail.
13. Opening a math or stat book and seeing a page full of problems.
14. Getting ready to study for a math test.
15. Being given a pop quiz in a math class.
Numerical anxiety
16. Reading a cash register receipt after your purchase.
17. Being given a set of numerical problems involving addition to solve on paper.
18. Being given a set of subtraction problems to solve.
19. Being given a set of multiplication problems to solve.
20. Being given a set of division problems to solve.
Math course anxiety
21. Buying a math textbook.
22. Watching a teacher work on an algebraic equation on the blackboard.
23. Signing up for a math course.
24. Listening to another student explain a math formula.
25. Walking into a math class.

Table 6 The original 25 RMARS items, with factor labels.

B Originally developed items and factors

- Explaining my thought process to my math instructor.
- Being asked a question by my math instructor.
- Explaining my thought process to a peer in my math class.
- Working in a group to solve a math problem.
- Working on a math problem for which I am not sure where to start.
- Being asked to support my algebraic reasoning with a picture.
- Explaining my thought process to my whole math class.

Table 7 Seven new items designed in (White & Visscher, 2015) to supplement the RMARS with a few active and interactive classroom situations.

FACTOR 2: Explanation Anxiety (4.01)	
Explaining my thought process to my math instructor.	0.97
Being asked a question by my math instructor.	0.92
Explaining my thought process to a peer in my math class.	0.84
Explaining my thought process to my whole math class.	0.77
†Being asked to support my algebraic reasoning with a picture.	0.41
†Working on a math problem for which I am not sure where to start.	0.32
FACTOR 3: Problem Solving Anxiety (3.74)	
Picking up math textbook to begin a difficult reading assignment.	0.94
∗Being given homework assignments of many difficult problems that are due the next class meeting.	0.88
∗Picking up math textbook to begin working on a homework assignment.	0.75
Working in a group to solve a math problem.	0.54
†Working on a math problem for which I am not sure where to start.	0.43
†Being asked to support my algebraic reasoning with a picture.	0.41
∗Realizing you have to take a certain number of math classes to fulfill requirements.	0.34
∗Opening a math or stat book and seeing a page full of problems.	0.33

Table 8 Two new factors from the RMARS + original 7 items, with eigenvalues and loadings. Items that load on both factors are marked with a †. Items from the RMARS instrument are marked with an ∗. The first factor in this analysis was Math Test Anxiety (eigenvalue 6.74), the fourth factor was Numerical Task Anxiety (eigenvalue 3.51), and the fifth factor was Math Course Anxiety (eigenvalue 3.45).

C Interaction of RMARS items with new factors

Reassigned to Problem Solving Anxiety
6. Being given homework assignments of many difficult problems that are due the next class meeting.
11. Picking up math textbook to begin a difficult reading assignment.
13. Opening a math or stat book and seeing a page full of problems.
Items without a significant loading
5. Picking up math textbook to begin working on a homework assignment.
10. Realizing you have to take a certain number of math classes to fulfill requirements.
16. Reading a cash register receipt after your purchase.
21. Buying a math textbook.
22. Watching a teacher work on an algebraic equation on the blackboard.
23. Signing up for a math course.
24. Listening to another student explain a math formula.
25. Walking into a math class.

Table 9 RMARS items that loaded onto a different factor than originally reported in an exploratory factor analysis of our data for the RMARS+MEARS instrument. Items that had no factor loading exceeding .50 are listed as not having a significant loading.